MATH 211
Calculus II Integral Calculus

Module 1 Integration

Topic 1: Antiderivatives

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Antiderivatives in General

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Commonly Used Antiderivatives

TABLE 4.2 Antiderivative formulas, k a nonzero constant

	Function	General antiderivative
1.	x^n	$\frac{1}{n+1}x^{n+1} + C, \quad n \neq -1$
2.	$\sin kx$	$-\frac{1}{k}\cos kx + C$
3.	$\cos kx$	$\frac{1}{k}\sin kx + C$
4.	$\sec^2 kx$	$\frac{1}{k}\tan kx + C$
5.	$\csc^2 kx$	$-\frac{1}{k}\cot kx + C$
6.	$\sec kx \tan kx$	$\frac{1}{k}\sec kx + C$
7.	$\csc kx \cot kx$	$-\frac{1}{k}\csc kx + C$
8.	e^{kx}	$\frac{1}{k}e^{kx} + C$
9.	$\frac{1}{x}$	$\ln \lvert x \rvert + C, \quad x \neq 0$
10.	$\frac{1}{\sqrt{1-k^2x^2}}$	$\frac{1}{k}\sin^{-1} kx + C$
11.	$\frac{1}{1+k^2x^2}$	$\frac{1}{k}\tan^{-1} kx + C$
12.	$\frac{1}{x\sqrt{k^2x^2-1}}$	$\sec^{-1} kx + C, \quad kx > 1$
13.	a^{kx}	$\left(\frac{1}{k \ln a}\right)a^{kx} + C, \quad a > 0, \ a \neq 1$

Linearity Rules

TABLE 4.3 Antiderivative linearity rules

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1. *Constant Multiple Rule*:	$kf(x)$	$kF(x) + C$, k a constant
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3. *Sum or Difference Rule*:	$f(x) \pm g(x)$	$F(x) \pm G(x) + C$

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$$\int f(x)\,dx.$$

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Example. Evaluate $\int (\sin x + 5x^4 - 2)dx$.

Challenge Problem

Evaluate $\int (6\sin 3x + x^2 - 12e^{2x})dx.$