MATH 211
Calculus II Integral Calculus

Module 1 Integration

Topic 2: Area and Estimating with Finite Sums

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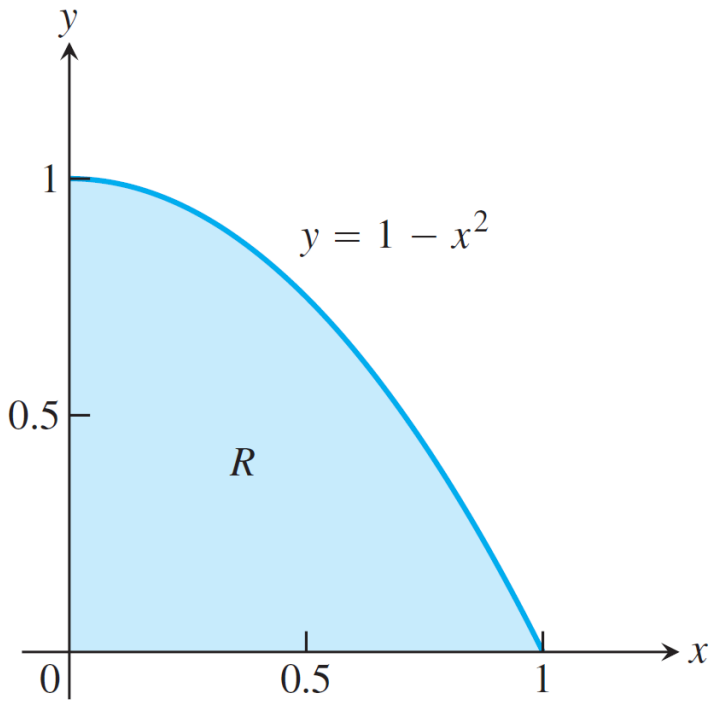
Estimating Are: An Example

Consider the region bounded by x-axis, y-axis and $y = 1 - x^2$. What is its area?



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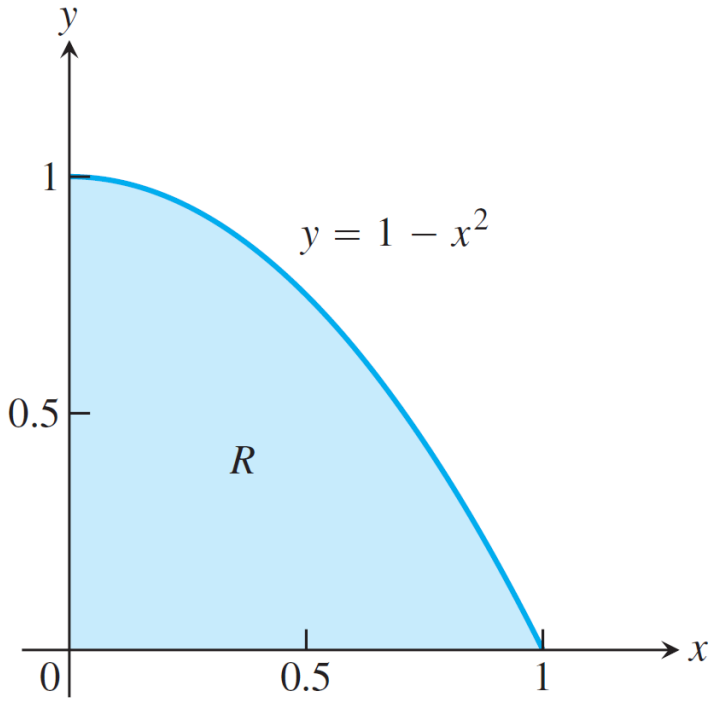

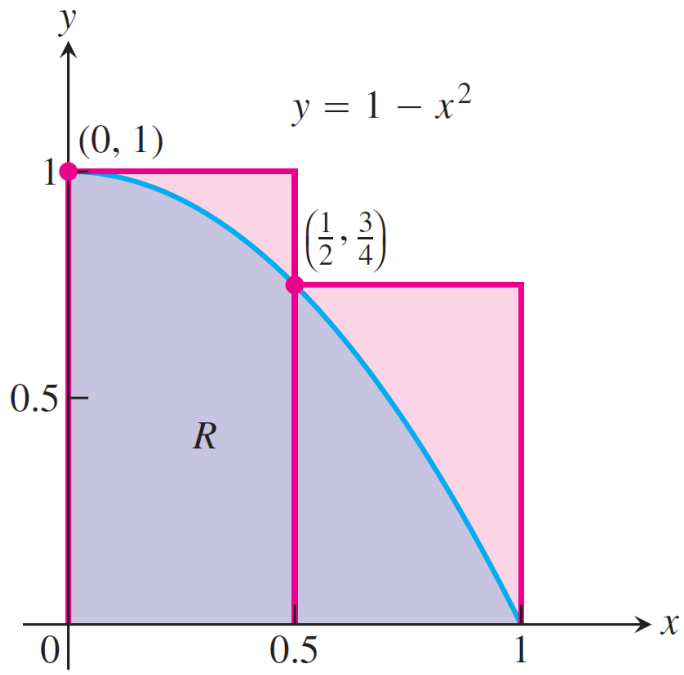

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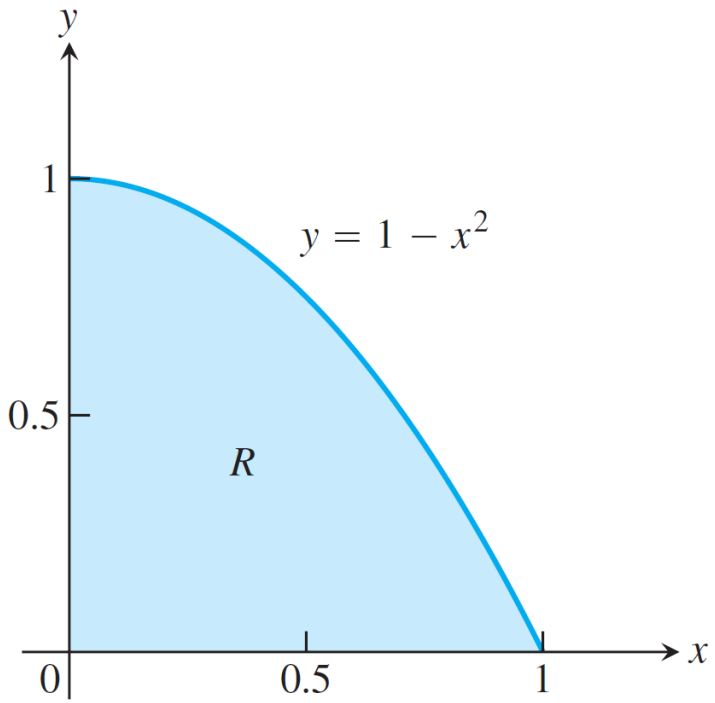
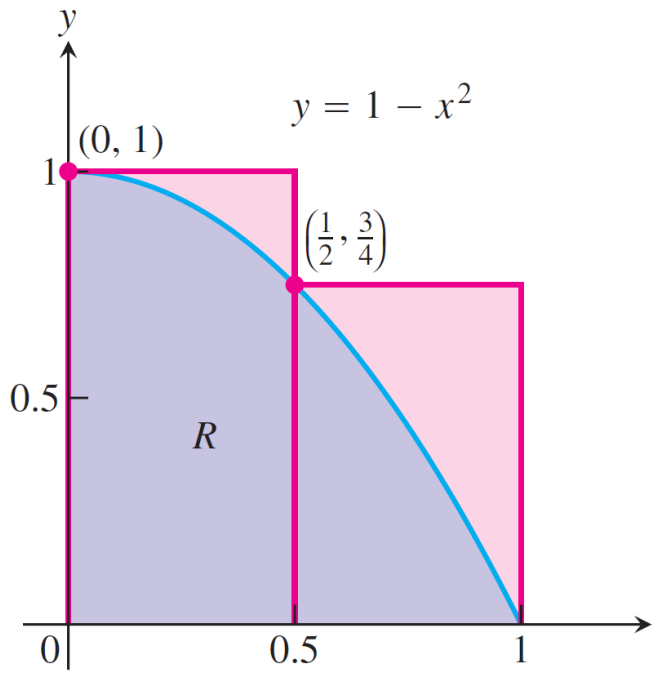
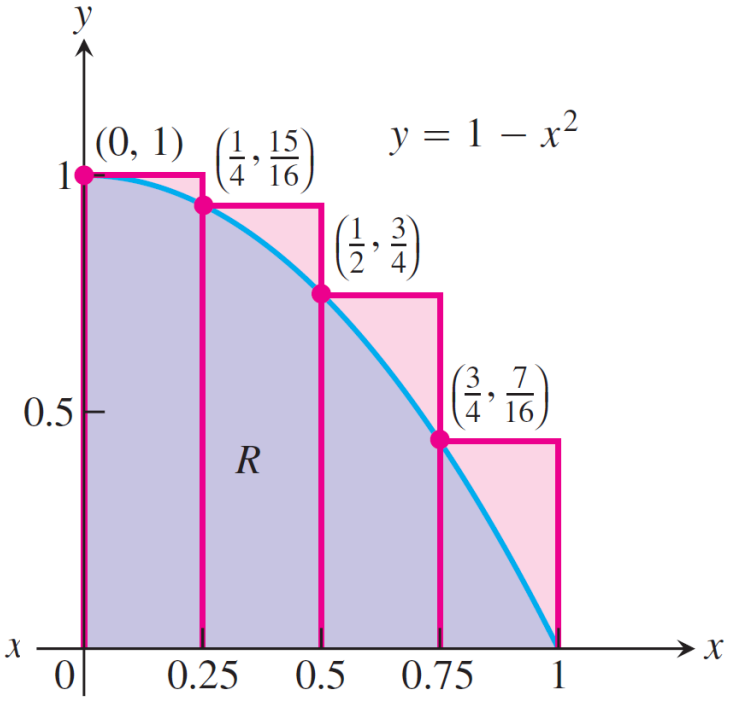
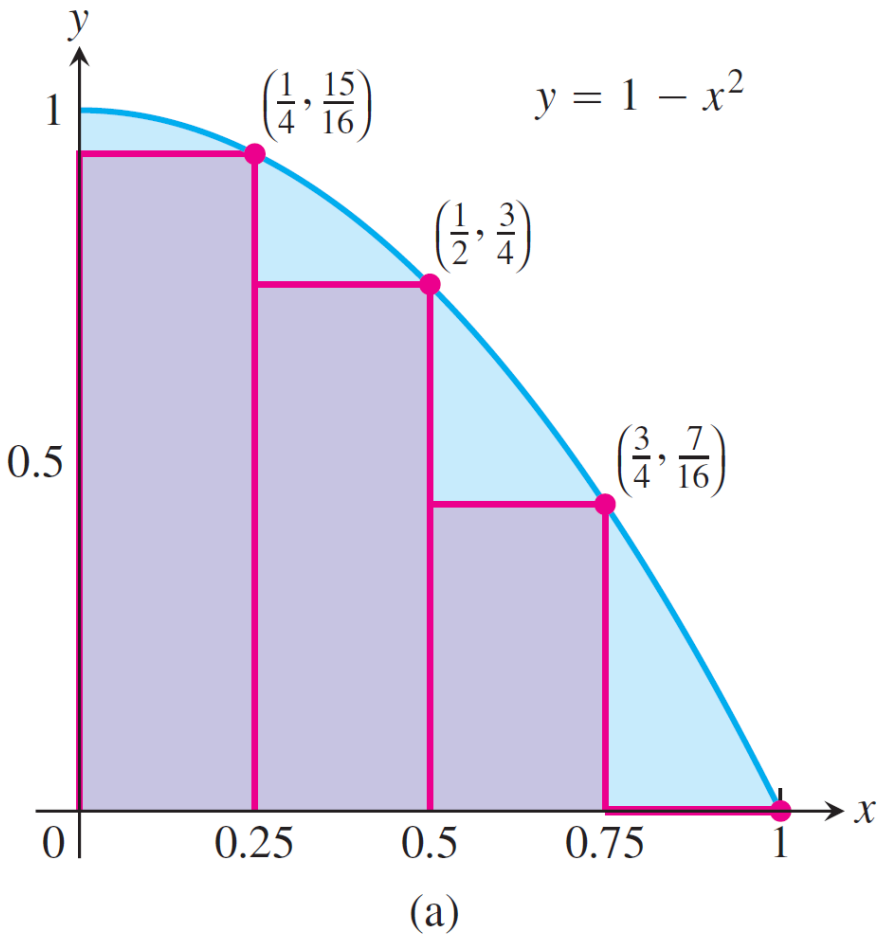




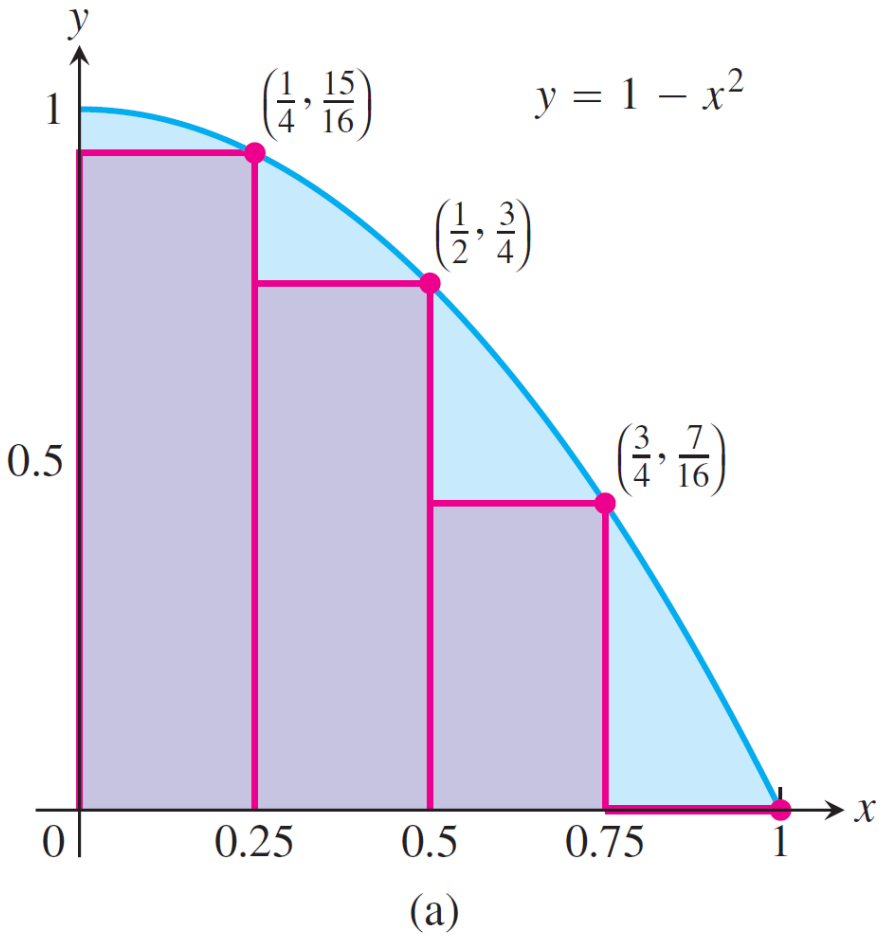


Other Approaches

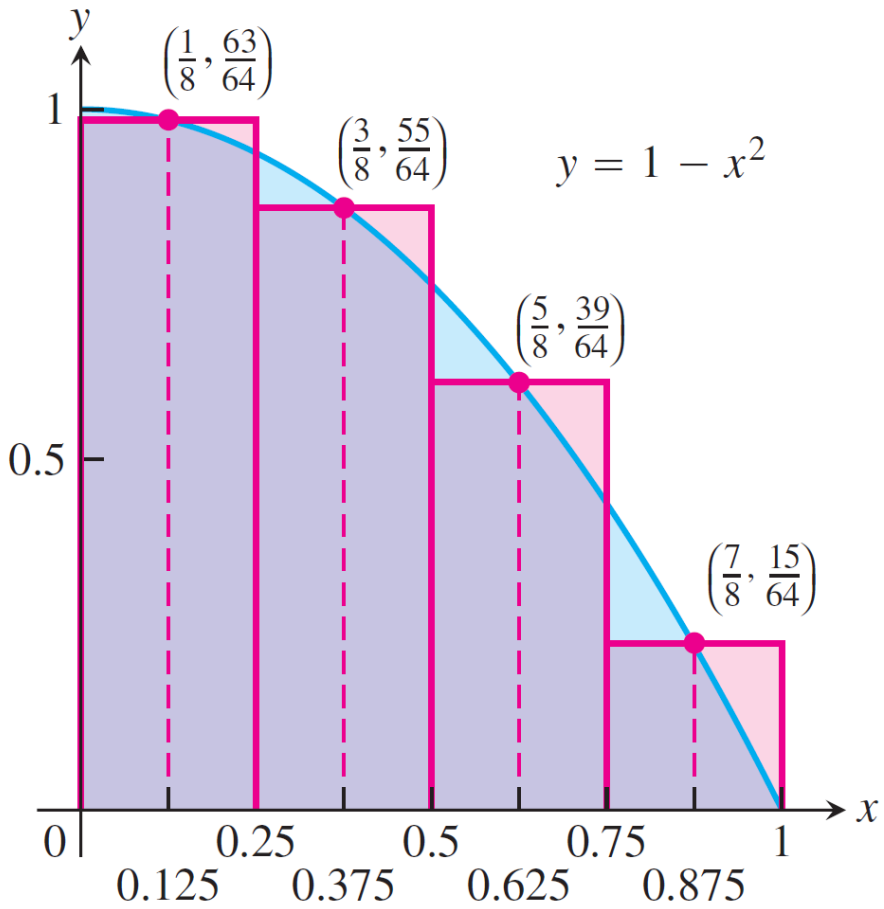


(a)

Other Approaches

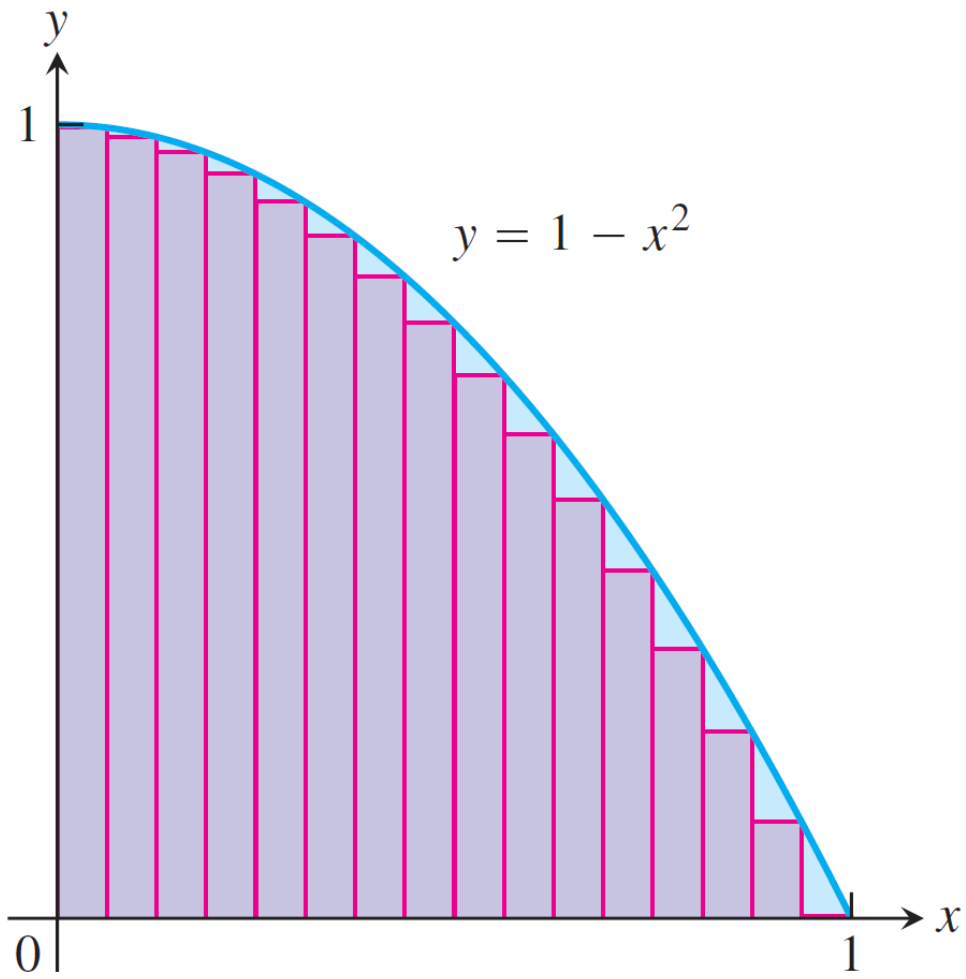


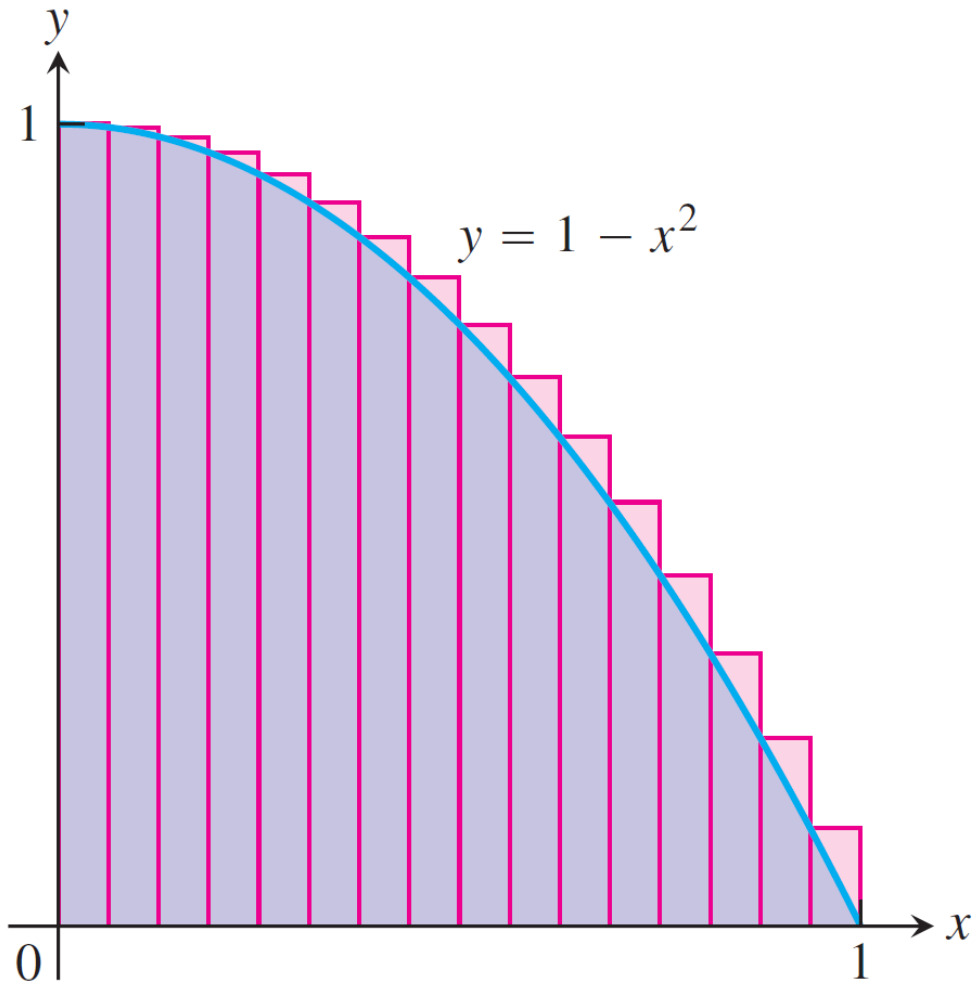
(a)



(b)

If we make narrower and narrower cuts...

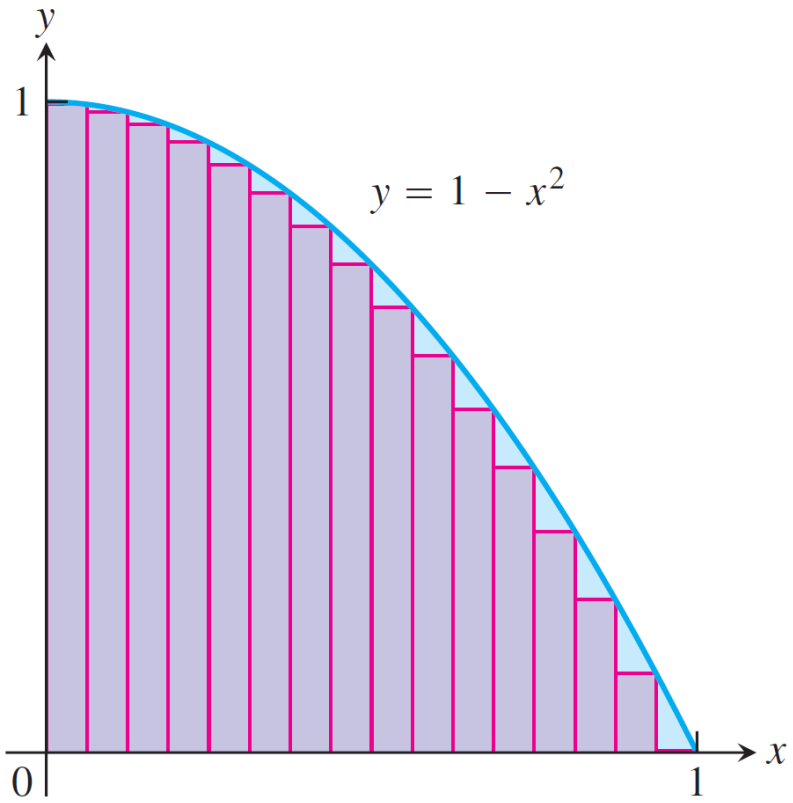




Challenge Problem

1. Estimate the area by making 5 equal “cuts” and use the left endpoint. Use a calculator or a spreadsheet on your computer.

2. How about 500 equal cuts?



A Distance-Velocity Problem

We know that when the velocity is constant,

$$distance = velocity \times time.$$

What if the velocity is NOT constant?

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Example. The velocity function of a projectile fired straight into the air is $f(t) = 160 - 9.8t$ m/sec. Estimate how far the projectile rises during the first 3 sec.

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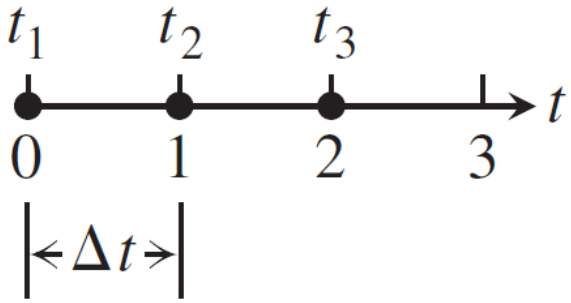
Estimate 1. Divide the 3 seconds into 3 equal time intervals.



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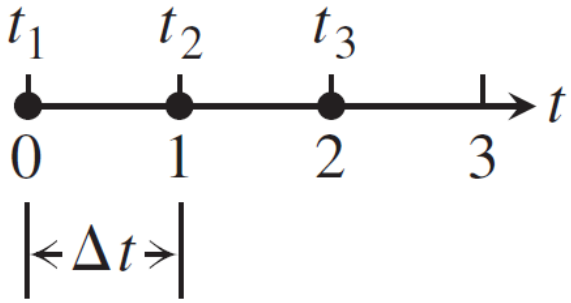

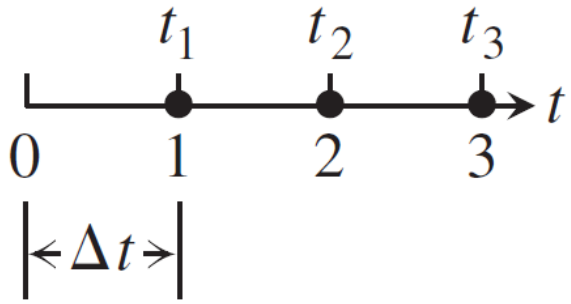

$f(t) = 160 - 9.8t$ m/sec

Estimate 2. Divide the 3 seconds into 6 equal time intervals.