

harm is done other than conveying the incorrect impression that “backwards proofs” are actual proofs, which they are not. By contrast, in real analysis it is crucial not to confuse “backwards proofs” with real proofs. For example, as we will see very clearly in Example 3.2.3, the  $\varepsilon$ - $\delta$  proofs discussed in Section 3.2 often require first some scratch work that is “backwards,” and then a rather different-looking proof that is “forwards.” Hence, it is very important in the proofs that you write for the exercises in this book that you distinguish between how you think of a proof, which can be any combination of “backwards,” “forwards” and anything else, and how you write the final draft of the proof, which must be very precise in going from what we assume to what we want to prove.

A few additional points about writing mathematical proofs are the following:

- Strategize the outline of a proof before working out the details; the outline of a proof is determined by what is being proved, not by what is hypothesized.
- Use definitions precisely as stated.
- Do not omit steps in proofs; when in doubt, prove it.
- Justify each step in a proof, citing the appropriate results from the text as needed.
- If a step in a proof is skipped, for example because it is very similar to a previous step, state explicitly that that is the case.
- Use correct grammar, including full sentences and proper punctuation.
- Use “=” signs properly.
- Proofs should stand on their own; check your proofs by reading them as if they were written by someone else.

See [Blo10, Section 2.6], [Gil87], [Hig98], [KLR89] and [SHSD73] for further discussion of writing mathematics.

The bottom line is to write your proofs very carefully, because doing so will help you learn the material in this book.