

# **Discrete Mathematics**

## **Counting and Probability**

Pangyen Weng, Ph.D  
Metropolitan State University



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# Bijection and the k-to-1 Rule

## Bijection Rule

Let  $S$  and  $T$  be two finite sets. If there is a bijection (a.k.a. one-to-one correspondence) from  $S$  to  $T$ , then  $|S| = |T|$ .

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**Example.**  $A = \{x, y, z\}$ .

1. Establish a 1-to-1 correspondence between  $P(A)$  and  $\{0,1\}^3$ .
2. Count  $P(A)$  by counting  $\{0,1\}^3$ .

## The $k$ -to-1 Rule

$f: X \rightarrow Y$  is a  $k$ -to-1 correspondence if for every  $y \in Y$ , there are exactly  $k$  different  $x \in X$  such that  $f(x) = y$ . In that case,  $|Y| = \frac{|X|}{k}$ , or  $|X| = k|Y|$ .

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**Example.**  $X = \{0,1,2,3,4,5\}$ ,  $Y = \{0,1\}$  and  $y = f(x) = \left\lfloor \frac{x}{3} \right\rfloor$ .