

Discrete Mathematics

Counting and Probability

Pangyen Weng, Ph.D
Metropolitan State University

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Conditional Probability

An Example

50 college students were surveyed, 20 of them are seniors.

- 12 of the 20 seniors are heavy coffee-drinkers.
- 13 of the other students are heavy coffee-drinkers.

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50 college students were surveyed, 20 of them are seniors.

- 12 of the 20 seniors are heavy coffee-drinkers.
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Answer the following questions.

1. ___% of ALL 50 students are heavy coffee-drinkers?
2. ___% of seniors are heavy coffee-drinkers?
3. ___% of the other students are heavy coffee-drinkers?
4. ___% of the heavy coffee drinkers are seniors?
5. Students in which group is more likely to be heavy coffee-drinkers?
Seniors **Other students** **No difference**

Definition

Let E and F be two events. Consider ONLY among the outcomes of F , the probability for E to happen is called the **conditional probability** of E given F , denoted by $p(E|F)$.

This probability is calculated by

$$p(E|F) = \frac{p(E \cap F)}{p(F)}$$

- › E is the event of interest.
 F is the condition.

Example

In a group of 50 students, 40% are senior and 24% are heavy coffee-drinking seniors. We would like to find out the percentage of heavy coffee-drinkers among seniors.

1. What is the event of interest E ?
2. What is the condition F ?
3. What is $p(E|F)$?

Conditional Probability for Uniform Distributions

For uniform distributions, $p(E|F) = |E \cap F|/|F|$.

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Example. Roll two dice, but one die at a time. If the first die gets a 5, what's the probability of getting a total of 10 or more?

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Exercise

Roll two dice consecutively. If the total is 10 or more, what is the probability that the first die gets a 5?

Theorem

Let E and F be two events. Then

$$p(E \cap F) = p(F) \cdot p(E|F) = p(E) \cdot p(F|E)$$

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Example. 7% of all students at Metro State are lefties. Of all the lefties, 20% are seniors. What's the probability that a randomly chosen student is a senior and a leftie?