Discrete Mathematics Counting and Probability

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Bayes' Theorem Part I

Events Under Conditions

Consider two events E and F in a sample space S. Also assume that a study is conducted in a way that

- The sample space is divided into two sets: F and $\bar{F}$.
- Within each condition, event E is studied.
- We call E the **event of interest** and F the **condition.**

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Example. Let S be the set of all the voters in St. Paul.

$$F = \text{the set of all voters 40 years old or under.}$$
$$E = \text{the set of all voters supporting Candidate X.}$$

Describe the study.

What to Expect

If a study is conducted with event E and condition F, we expect to measure:

1. $P(F)$. So $P(\bar{F}) = 1 - P(F)$.
2. Within F, $P(E|F)$. So $P(\bar{E}|F) = 1 - P(E|F)$.
3. Within $\bar{F}$, $P(E|\bar{F})$. So $P(\bar{E}|\bar{F}) = 1 - P(E|\bar{F})$.

Example

Let S be the set of all the voters in St. Paul. Assume that 32% of the voters are 40 or under. Also assume that 25% of voters 40 or under support Candidate X, but 55% of voters above 40 support Candidate X. Identify all 6 probabilities above.

Probability of Event E

Since the probability of E is only measured within F and $\bar{F}$, we only have $P(E|F)$ and $P(E|\bar{F})$.

$$P(E \cap F) = P(F) \cdot P(E|F)$$

$$P(E \cap \bar{F}) = P(\bar{F}) \cdot P(E|\bar{F})$$

So, $P(E) = P(F) \cdot P(E|F) + P(\bar{F}) \cdot P(E|\bar{F})$.

Example

In a high school, 25% of the students are seniors. Of the seniors, 6% went to last Friday's basketball game. Of the other students, 10% went to the game. How many percent of the entire student body went to the game?

1. Identify E and F.
2. Solve the question.