Discrete Mathematics Counting and Probability

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Bayes' Theorem Part II

Bayes' Theorem

Let E and F be events that have nonzero probability. Suppose a study uses F as the condition and E as the event of interest.

Then we can calculate the probability of F under condition E:

$$p(F|E) = \frac{p(E \cap F)}{p(E)} = \frac{p(F) \cdot p(E|F)}{p(F) \cdot p(E|F) + p(\bar{F}) \cdot p(E|\bar{F})}$$

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Remark. This theorem reverses the event and the condition.

Example 1

In a high school, 25% of the students are seniors. Of the seniors, 6% went to last Friday's basketball game. Of the other students, 10% went to the game.

1. How many percent of the entire student body went to the game?
2. Of those who went to the game, how many percent were seniors?

Example 2

Suppose that 1 out of every 1000 computer chips produced by a chip manufacturer has a defect. The manufacturer has developed a method to test the chips, but the test is not perfect.

- If the chip has a defect, the test will discover the defect with probability 0.99.
- If the chip does not have a defect, the test will report that it does have a defect with probability 0.02.

If the outcome of a particular test indicates that there is a defect, what is the likelihood that the chip is actually faulty?

Exercise

One out of every 10,000 people has a particular genetic disease. A test has been developed for the disease that is very accurate but has some likelihood of error.

- When a person with the disease is tested, there is a 0.01 probability that the test says he does NOT have the disease.
- When a person without the disease is tested, there is a 0.005 probability that the test says he has the disease.

If a person tests positive for the disease, what is the probability that this person indeed has the disease?