Discrete Mathematics
Relations and Directed Graphs

Pangyen Weng, Ph.D
Metropolitan State University

Composition of Relations

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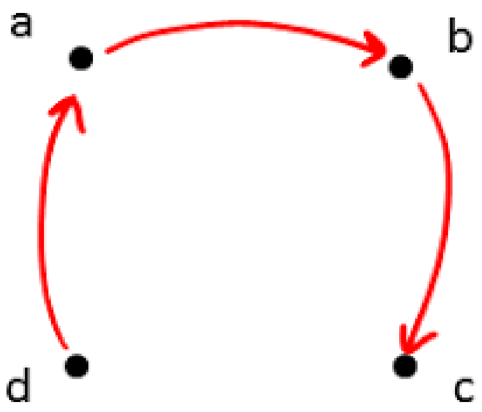
Let R and S be two relations on a set A. The ***composition*** of R and S, denoted $S \circ R$, is defined by

$$S \circ R = \{(a, c): aRb \text{ and } bSc \text{ for some } b \in A\}.$$

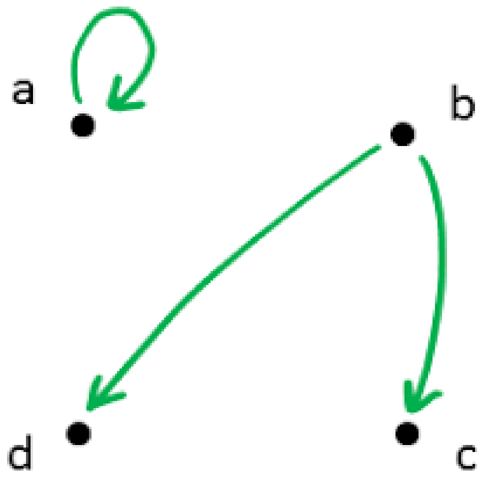
In other words, $a \xrightarrow{R} b \xrightarrow{S} c$, or $a(S \circ R)c$.

Example

Find $S \circ R$.



R



S

Powers of Graphs

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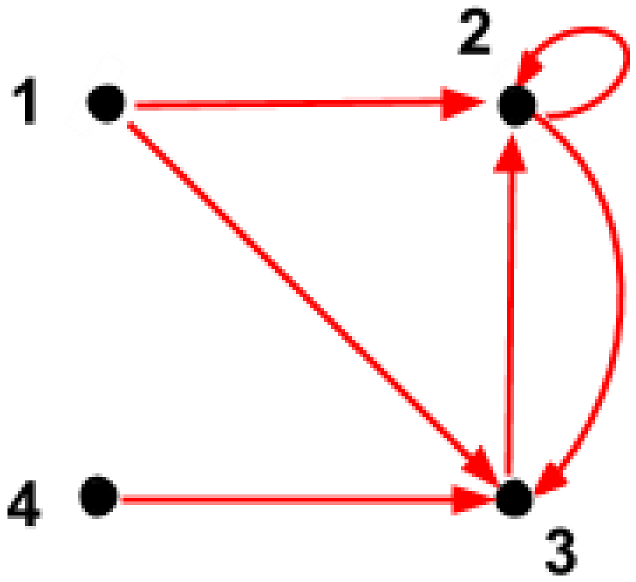
Example. Graph G^2 and G^3.



The Transitive Closure

Definition. $G^+ = G \cup G^2 \cup G^3 \cup G^4 \cup \cdots$

In reality, we only need to consider up to as many vertices:

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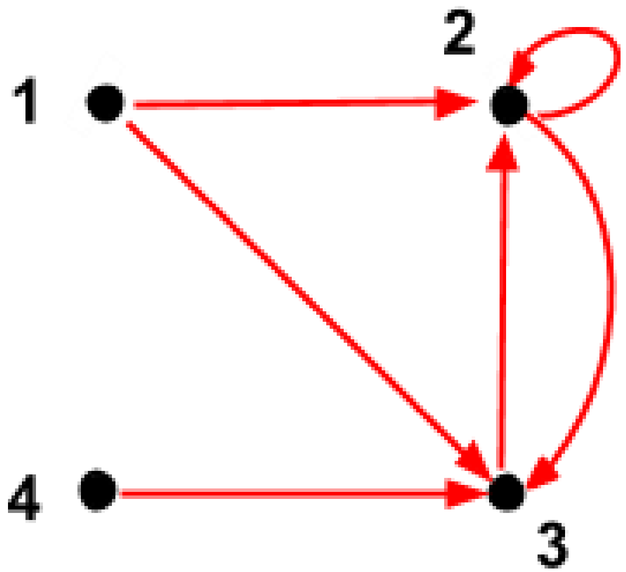
Example. Graph G^+.



Example

True or false?

1. $(a, b) \in G^2$
2. $(b, e) \in G^3$
3. $(g, g) \in G^3$
4. $(g, g) \in G^4$
5. $(f, f) \in G^+$
6. $(a, a) \in G^+$