

**Topic 01 Plane curves: Introduction, derivatives and arclengths**

What is a parametric curve?

Give an example of parametric curve and sketch it.

Consider the parametric curve  $x = 4 \cos t$ ,  $y = 3 \sin t$ ,  $0 \leq t \leq \pi/2$ .

1. Find its Cartesian equation.
2. Sketch the curve with its initial point, terminal point and direction.

Find parametric equations and a parameter interval for  $x^2 + y^2 = 4$ :

1. Beginning at  $(2, 0)$  clockwise for half a circle.
2. Beginning at  $(0, 2)$  counterclockwise for two circles.

Parametrize the line segment that goes...

1. from  $(3, 2)$  to  $(-1, 5)$ .
2. from  $(-1, 0)$  to  $(-1, 7)$ .

Use a graphing device and graph  $x = t - \sin t$  and  $y = 1 - \cos t$  for at least three different parameter intervals.

Find the length of the curve:  $x = t^2/2$ ,  $y = (2t + 1)^{3/2}/3$ ,  $0 \leq t \leq 4$ .

Find  $dy/dx$  and  $dy^2/dx^2$  for  $x = 2t^2 + 4t + 1$  and  $y = t^4$ .

## Self Assessment

0 = clueless

1 = vaguely understand what the topic is about

2 = can explain the topic to another person

3 = can perform direct calculations

4 = can apply in problem-solving

Circle your level of comprehension about the following.

- |  |   |   |   |   |   |
|--|---|---|---|---|---|
| • The parametric equations of curves.                        | 0 | 1 | 2 | 3 | 4 |
| • Converting parametric curves to Cartesian ones.            | 0 | 1 | 2 | 3 | 4 |
| • The difference between parametric and Cartesian curves.    | 0 | 1 | 2 |   |   |
| • The first and the second derivatives of parametric curves. | 0 | 1 | 2 | 3 | 4 |
| • The arclength of a curve.                                  | 0 | 1 | 2 | 3 |   |

**Your own notes, remarks and important information:**



**Topic 02 Vectors, dot products and cross products**

Let  $\mathbf{u} = \langle 1, -2, 2 \rangle$  and  $\mathbf{v} = \langle 4, 3, 1 \rangle$ .

1. Calculate  $3\mathbf{u} - 2\mathbf{v}$ .
2. If the initial point of  $\mathbf{u}$  is at  $(-3, 0, -4)$  where is its terminal point?
3. Rewrite  $\mathbf{v}$  in terms of  $\mathbf{i}$ ,  $\mathbf{j}$  and  $\mathbf{k}$ .
4. Find the length and the direction of  $\mathbf{u}$ .

Let  $\mathbf{u} = \langle 1, -2, 2 \rangle$ ,  $\mathbf{v} = \langle 4, 3, 1 \rangle$  and  $\mathbf{w} = \langle -3, 4, 0 \rangle$ .

1. Are  $\mathbf{u}$  and  $\mathbf{u}$  perpendicular to each other? Explain your answer.

2. Find the angle between  $\mathbf{u}$  and  $\mathbf{w}$ .

3. Calculate  $\mathbf{u} \times \mathbf{v}$ .



True or false? Choose and explain.

- |  |      |       |
|--|------|-------|
| 1. The dot product of two vectors is a scalar (aka number).  | True | False |
| 2. Two vectors are orthogonal (aka perpendicular to each other) if their dot products equals 0, and vice versa.    | True | False |
| 3. The cross product of two vectors is a scalar.   | True | False |
| 4. We consider cross products for five-dimensional vectors.  | True | False |
| 5. For any vectors $\mathbf{u}$ and $\mathbf{v}$ , $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$ .   | True | False |
| 6. For any vectors $\mathbf{u}$ and $\mathbf{v}$ , $\mathbf{u} \times \mathbf{v} = \mathbf{v} \times \mathbf{u}$ . | True | False |

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Circle your level of comprehension about the following.

- |   |   |   |   |   |   |
|---|---|---|---|---|---|
| • Basic arithmetic of vectors (addition, subtraction, etc.) | 0 | 1 | 2 | 3 | 4 |
| • The dot products of vectors.                              | 0 | 1 | 2 | 3 | 4 |
| • Using dot products to show orthogonal vectors.            | 0 | 1 | 2 | 3 | 4 |
| • Using dot products to find the angle between vectors.     | 0 | 1 | 2 | 3 | 4 |
| • The cross product of vectors.                             | 0 | 1 | 2 | 3 | 4 |

**Your own notes, remarks and important information:**

**Topic 03 Line and planes in space**

1. Parametrize the line that passes through points  $P(1, 2, -3)$  and  $Q(0, -4, 1)$ .
2. Parametrize the *line segment* that starts at  $P(1, 2, -3)$  and ends at  $Q(0, -4, 1)$ .
3. Find the parametric equations for the line that passes through  $P(0, -5, -3)$  and in the direction of  $2\mathbf{j} - 7\mathbf{k}$ .
4. Find an equation for the plane that passes through  $(3, -2, 4)$  and is normal to  $\mathbf{i} + \mathbf{j} - 4\mathbf{k}$ .

5. Find an equation for the plane that passes through the three points  $P(1, 0, 5)$ ,  $Q(0, 0, 3)$  and  $R(-2, 0, 0)$ .

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Circle your level of comprehension about the following.

- |  |   |   |   |   |   |
|--|---|---|---|---|---|
| • The parametric equations of lines.         | 0 | 1 | 2 | 3 | 4 |
| • Finding the parametric equations of lines. | 0 | 1 | 2 | 3 | 4 |
| • The equations of planes.                   | 0 | 1 | 2 | 3 | 4 |
| • Finding the equations of plane.            | 0 | 1 | 2 | 3 | 4 |

**Your own notes, remarks and important information:**

**Topic 04 Cylinders and quadric surfaces**

Use a graphing device and sketch the following.

1.  $x^2 + y^2 = z^2 + 1$

2.  $x^2 + y^2 = z^2$

3.  $x^2 + y^2 = z^2 - 1$

4.  $x^2 + y^2 = 1 - z^2$

5.  $x^2 - y^2 = z$

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Circle your level of comprehension about the following.

- The quadric equations and their surfaces. 0    1    2
- Using software to sketch quadric surfaces. 0    1    2

**Your own notes, remarks and important information:**