

Topic 05 Curves in Space and Their Tangents

What is a position vector $\mathbf{r}(t)$? What are its *component functions* and what is the curve traced by \mathbf{r} ?

Evaluate: $\lim_{t \rightarrow 0} \left\langle \tan(1 - t^2), \frac{e^t - 1}{t}, \frac{\sin t}{2t} \right\rangle$.

Is $\mathbf{r}(t) = (t^2)\mathbf{i} + (\sec t)\mathbf{j} - (5 \sin t)\mathbf{k}$ continuous at $t = \pi/2$? Explain your answer.

$\mathbf{r} = (\cos 2t)\mathbf{i} + (\sin 2t)\mathbf{j} + (5t)\mathbf{k}$. Find its velocity, speed and acceleration at $t = \pi/2$.

Self Assessment

0 = clueless

1 = vaguely understand what the topic is about

2 = can explain the topic to another person

3 = can perform direct calculations

4 = can apply in problem-solving

Circle your level of comprehension about the following.

- | | | | | | |
|--|---|---|---|---|---|
| • What is a vector-valued function? | 0 | 1 | 2 | | |
| • Position vectors $\mathbf{r}(t)$ and parametric curves. | 0 | 1 | 2 | | |
| • The limit and the derivative of $\mathbf{r}(t)$. | 0 | 1 | 2 | 3 | 4 |
| • Calculating the velocity and the acceleration of $\mathbf{r}(t)$. | 0 | 1 | 2 | 3 | 4 |

Your own notes, remarks and important information:

Topic 06 Integrals of Vector Functions

How do you evaluate *indefinite integrals* for vector functions?

Evaluate

$$\int \left((t\sqrt{1+t^2})\mathbf{i} + (6\sqrt[3]{t})\mathbf{j} + (e^t \sin(e^t + 1))\mathbf{k} \right) dt.$$

How do you evaluate *definite integrals* for vector functions?

Evaluate

$$\int_0^1 \left((t\sqrt{1+t^2})\mathbf{i} + (6\sqrt[3]{t})\mathbf{j} + (e^t \sin(e^t + 1))\mathbf{k} \right) dt.$$

Suppose we do not know the position \mathbf{r} of a moving object, but we do know its acceleration is $\mathbf{a}(t) = (9 \sin 3t)\mathbf{i} - (9 \cos 3t)\mathbf{j}$ and that initially the object departed from the point $(1, 2, 0)$ with velocity $3\mathbf{j} + \mathbf{k}$. Solve for \mathbf{r} .

Write down the formula(s) for the ideal projectile motion. Write down at least 3 conclusions that can be derived from the result of this analysis.

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- | | | | | | |
|---|---|---|---|---|---|
| 1. Integration $\mathbf{r}(t)$. | 0 | 1 | 2 | 3 | 4 |
| 2. Finding $\mathbf{r}(t)$ from its initial position, initial velocity and its acceleration vector. | 0 | 1 | 2 | 3 | 4 |

Topic 07 Arclength

What is the formula for calculating the arclength of $\mathbf{r}(t)$ from $t = a$ to $t = b$?

Set up the integral for the arclength of $\mathbf{r} = e^t\mathbf{i} + 2t\mathbf{j} + t^2\mathbf{k}$ from $t = 0$ and $t = 2$. This is not an integral that can be calculated by hand, but find a way to estimate it.

Find the unit tangent vector for $\mathbf{r} = e^t\mathbf{i} + 2t\mathbf{j} + t^2\mathbf{k}$ at $t = 1$.

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Circle your level of comprehension about the following.

- | | | | | |
|---|---|---|---|---|
| • Computing the arclength of a curve. | 0 | 1 | 2 | 3 |
| • Computing the unit tangent vector of a curve. | 0 | 1 | 2 | 3 |

Your own notes, remarks and important information:

Topic 08 Curvature and Normal Vectors

What does curvature measure? What is the formula for calculating the curvature of a parametric curve?

Find the curvature of $\mathbf{r} = t\mathbf{i} + 2t\mathbf{j} + t^2\mathbf{k}$ at $t = 1$.

What is the principal normal vector \mathbf{N} for a parametric curve? What is the formula for calculating it?

Find the principal normal vector \mathbf{N} of $\mathbf{r} = t\mathbf{i} + 2t\mathbf{j} + t^2\mathbf{k}$ at $t = 1$.

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Circle your level of comprehension about the following.

- | | | | | |
|--|---|---|---|---|
| • Curvature. | 0 | 1 | 2 | 3 |
| • Principal normal vector \mathbf{N} . | 0 | 1 | 2 | 3 |

Your own notes, remarks and important information:

Topic 09 Tangential and Normal Components Normal Components of Acceleration

What is a TNB-frame? Write down the formulas for \mathbf{T} , \mathbf{N} and \mathbf{B} .

What is a torsion? Write down the formulas for the torsion for a parametric curve \mathbf{r} .

Consider for $\mathbf{r} = \langle \cos 3t, \sin 3t, 4t \rangle$ at $t = 0$.

1. Find \mathbf{T} , \mathbf{N} and \mathbf{B} .
2. Find its torsion.

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Circle your level of comprehension about the following.

- The binormal vector \mathbf{B} .

0 1 2 3

- Torsion.

0 1 2 3

Your own notes, remarks and important information: