

# **Discrete Mathematics**

## **Induction and Recursion**

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# Summations of Special Sequences

# Arithmetic Sequences

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2. 95 is its \_\_\_\_\_th term.
3. What is the sum of its first 20 terms?

## Closed Forms of Geometric Sequences

Consider a geometric sequence with initial term be  $a_0$  and the common ratio  $r$ . Then

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