Discrete Mathematics
Induction and Recursion

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Examples.

1. $n^2 + 3n$ is an even number for all integers $n = 1, 2, 3, \ldots$
2. $n^2 - 2n - 1 > 0$ for all integers $n = 3, 4, 5, \ldots$

Principle of Mathematical Induction

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The principle:

1. There is an initial case $P(m)$ that is true.
2. Afterwards, we logically establish that every previous case implies the next. I.e.
$$P(k) \Rightarrow P(k+1), \text{for all } k \geq m.$$
3. So, $P(m) \Rightarrow P(m+1) \Rightarrow P(m+2) \Rightarrow \cdots \Rightarrow$ all integers that follow.

Example

Prove that for any $n \in \boldsymbol{N}$, $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.