

Topic 01 Propositions and Operations

What is a proposition?

What is a compound proposition?

What is a conjunction $p \wedge q$? Draw the truth table for $p \wedge q$.

What is a disjunction $p \vee q$? Draw its truth table.

What is an exclusive-or $p \oplus q$? Draw its truth table.

Topic 02 Evaluating Compound Propositions

$p = q = T$ and $r = F$. Evaluate $p \wedge \neg(q \oplus r)$.

Construct a truth table for $p \vee (q \wedge r)$.

Topic 03 Conditional Propositions

What is a conditional proposition $p \rightarrow q$? What do we call p and q ?

Draw the truth table for $p \rightarrow q$.

Consider the proposition: *If I like apples then I like apple pies.*

What are the converse, the inverse and the contrapositive?

The **converse**:

The **inverse**:

The **contrapositive**:

What is a biconditional proposition $p \leftrightarrow q$? Draw its truth table.

Topic 04 Logical Equivalence

What is a tautology? What is a contradiction?

What does it mean for two propositions to be *logically equivalent*?

Write down an example of two equivalent propositions.

Prove your example of equivalent propositions using their truth tables.

Prove this De Morgan's law: $\neg(p \wedge q) \equiv (\neg p) \vee (\neg q)$.

Topic 05 Predicates and Quantifiers

What is a predicate? Describe and make an example.

Why is it important to be aware of the domain of a predicate?

What is a **universal proposition**? What is the quantifier for universal propositions?

Make a universal proposition. Determine if it is true.

What is an **existential proposition**? What is the quantifier for existential propositions?

Make an existential proposition. Determine if it is true.

How do we use **counterexamples** and **examples** in quantified propositions?

Topic 06 Proving or Disproving Quantified Propositions

Make a universal proposition that is *true*. Prove it.

Make a universal proposition that is *false*. Disprove it.

Make an existential proposition that is *true*. Prove it.

Make an existential proposition that is *false*. Disprove it.

Topic 07 Nested Quantifiers What are the meanings of the following proposition? And what are their truth values? Here x and y are in $\{0, 1, 2\}$ and $P(x, y) : "x = y"$.

$$\forall x \forall y P(x, y)$$

$$\exists x \exists y P(x, y)$$

$$\exists y \forall x P(x, y)$$

$$\forall x \exists y P(x, y)$$

Sometimes altering the order of quantifiers would change the meaning (and the truth value) of a quantified proposition.

What kind of proposition changes its meaning when the order of its quantifiers is altered?

What kind of proposition remains the same when the order of its quantifiers is altered?

Topic 08 De Morgan's Laws

What are De Morgan's laws for quantified propositions?

Make an example of using De Morgan's laws.

What does negation do to quantifiers \forall and \exists ?

Remark. The negation of $p \rightarrow q$ is: _____

Topic 09 Mathematical Proofs

What is a theorem? What is a proof?

Make an example of a theorem and its proof.

What is *proof by exhaustion*?

Make an example of proof by exhaustion.

Topic 10 Direct Proof

What is a direct proof? Describe the method and procedure.

Define rational numbers and give some example.

Prove that if x is rational then $x + \frac{1}{2}$ is rational.

Prove that if x and y are rational then $x + y$ is rational.

Topics 11 More Examples of Direct Proof

Define odd numbers and even numbers.

Use definition to argue that 0 and 18 are even, and -9 is odd.

Prove that if n is even then $n^2 + 3n$ is even.

Prove that if n is odd then n^2 is odd.

Topic 12 Proof by Contraposition

What is proof by contraposition? Describe the method and procedure.

When do we consider using a proof by contraposition?

Prove that if $3n + 7$ is even then n is odd.

Prove that if x is irrational then $2x$ is irrational.

Topic 13 Proof by Contradiction

What is proof by contradiction? Describe the method and procedure.

When do we consider using a proof by contradiction?

Prove: If there are 26 people, then at least 3 of them were born in the same month.

Prove that $\sqrt{2}$ is an irrational number.