Which of these are propositions? What are the truth values of those that are propositions?

1. Would you like some coffee?
2. I would like coffee with cream and sugar.
3. Either Mom or Dad will pick me up.
4. There is no Chinese or Korean player on this basketball team.
5. What a beautiful day!
6. $3 + 5 = 7$.
7. Put on your jacket or you'll catch a cold.

Build a tree diagram for each of the following Cartesian products.

1. $\{$burger, sandwich$\} \times \{$coke, sprite, juice$\}$.

2. $\{$burger, sandwich$\} \times \{$coke, sprite, juice$\} \times \{$fries, chips$\}$.

3. $\{$True, False$\} \times \{$True, False$\} \times \{$True, False$\}$.

Evaluate the following.

1. $(T \vee F) \wedge F$

2. $(T \oplus \neg F) \rightarrow F$

Construct truth tables for the following.

1. $\neg(p \wedge q)$

2. $(\neg p) \vee (\neg q)$

Let p, q, and r be the propositions

$$p: \text{ Grizzly bears have been seen in the area.}$$
$$q: \text{ Hiking is safe on the trail.}$$
$$r: \text{ Berries are ripe along the trail.}$$

Write these propositions using p, q, and r and logical connectives (including negations).

1. Berries are ripe along the trail, but grizzly bears have not been seen in the area.
2. Grizzly bears have not been seen in the area and hiking on the trail is safe, but berries are ripe along the trail.
3. If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.
4. It is not safe to hike on the trail, but grizzly bears have not been seen in the area and the berries along the trail are ripe.
5. For hiking on the trail to be safe, it is necessary but not sufficient that berries not be ripe along the trail and for grizzly bears not to have been seen in the area.
6. Hiking is not safe on the trail whenever grizzly bears have been seen in the area and berries are ripe along the trail.

Determine if the following Propositions are logically equivalent.

1. $(\neg p) \rightarrow q$ and $p \vee q$.

2. $(p \oplus q)$ and $\neg(p \leftrightarrow q)$.

3. $(p \vee q) \wedge r$ and $(p \wedge r) \vee (q \wedge r)$.

Three professors are seated in a restaurant.

> The waitress: "Does everyone want coffee?"
>
> The first professor: "I do not know."
>
> The second professor: "I do not know."
>
> The third professor: "No, not everyone wants coffee."

The waitress comes back and gives coffee to the professors who want it. How did she figure out who wanted coffee?

On the island of knights and knaves created by Smullyan, knights always tell the truth and knaves always lie. You encounter two people, A and B. Determine, if possible, what A and B are if they address you in the ways described. If you cannot determine what these two people are, can you draw any conclusions?

1. A says ”At least one of us is a knave” and B says nothing.
2. A says ”The two of us are both knights” and B says ”A is a knave.”
3. A says ”I am a knave or B is a knight” and B says nothing.
4. Both A and B say ”I am a knight.”
5. A says ”We are both knaves” and B says nothing.

Show that $[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a tautology by using truth tables.

Show that $(p \to q) \to r$ and $p \to (q \to r)$ are not equivalent.

Let $P(x)$ be the statement "the word x contains the letter a." What are these truth values?

1. $P(\text{orange})$
2. $P(\text{lemon})$
3. $P(\text{true})$
4. $P(\text{false})$

For integers n, consider $Q(n)$ be the statement "$n \geq 2n$." Evaluate the following.

1. $Q(3)$.
2. $Q(-5)$.
3. $Q(0)$.
4. $\forall n\, Q(n)$.
5. $\exists n\, Q(n)$.

Express each of these statements using quantifiers. Then form the negation of the statement so that no negation is to the left of a quantifier. Next, express the negation in simple English. (Do not simply use the phrase ”It is not the case that.”)

1. All dogs have fleas.

2. There is a horse that can add.

3. Every koala can climb.

4. No monkey can speak French.

5. There exists a pig that can swim and catch fish.

Translate each of these quantifications into English and determine its truth value.

1. $\forall x \in \mathbf{R}\,(x^2 \neq -1)$

2. $\exists x \in \mathbf{Z}\,(x^2 = 2)$

3. $\forall x \in \mathbf{Z}\,(x^2 > 0)$

4. $\exists x \in \mathbf{Z}\,(x^2 = x)$

5. $\exists x \in \mathbf{Z}\,(x + 1 > x)$

Express each of these statements using predicates, quantifiers, logical connectives, and mathematical operators where the domain consists of all integers.

Example. The square of an odd number is an odd number.
(Solution 1). **Domain**: the set of all odd number. **Statement**: $\forall m\,(m^2 \text{ is an odd number.})$.
(Solution 2). **Domain**: the set of all integers. **Statement**: $\forall m\,\big((m \text{ is an odd number}) \to (m^2 \text{ is an odd number.})\big)$

1. The cube of an even number is even.

2. Half of any positive number is still positive.

3. For all integers n, if n^2 is even then n is even.

Express each of these statements using predicates, quantifiers, logical connectives, and mathematical operators where the domain consists of all integers.

Example. The product of two negative integers is positive.
(Solution 1). **Domain**: the set of all negative numbers. **Statement**: $\forall m \forall n\,(mn > 0)$.
(Solution 2). **Domain**: the set of all integers. **Statement**: $\forall m \forall n\ ((m < 0) \wedge (n < 0)) \rightarrow (mn > 0)$.

1. The average of two positive integers is positive.

2. The difference of two negative integers is not necessarily negative.

3. The absolute value of the sum of two integers does not exceed the sum of the absolute values of these integers.

Let $P(x, y)$ be the statement "Student x has taken class y," where the domain for x consists of all students in your class and the domain for y consists of all computer science courses at Metro State University. Express each of these quantifications in English.

1. $\exists x \exists y \, P(x, y)$

2. $\exists x \forall y \, P(x, y)$

3. $\forall x \exists y \, P(x, y)$

4. $\exists y \forall x \, P(x, y)$

5. $\forall y \exists x \, P(x, y)$

6. $\forall x \forall y \, P(x, y)$

Let $Q(x, y)$ be the statement "$x = y + 1$." If the domain for both variables consists of all integers, what are the truth values?

1. $Q(1, 1)$

2. $\forall y\, Q(1, y)$

3. $\exists x\, Q(x, 2)$

4. $\exists x \exists y\, Q(x, y)$

5. $\forall x \exists y\, Q(x, y)$

6. $\exists y \forall x\, Q(x, y)$

Determine the truth value of each of these statements if the domain of each variable consists of all real numbers.

1. $\forall x \exists y\,(x = y^2)$

2. $\exists x \forall y\,(xy = 0)$

3. $\exists x \forall y\,(y \neq 0 \rightarrow xy = 1)$

4. $\forall x \exists y\,(x + y = 1)$

5. $\forall x \forall y \exists z\,(z = (x + y)/2)$

Prove for all integers m: *if m is odd then $3m + 7$ is even.*

1. What proof would you choose? **direct** **indirect**
   Why?

2. What do you assume?

3. What conclusion do you wish to draw?

4. Complete the proof.

Prove that *the square of a rational number is rational.*

1. Rephrase the proposition in the conditional form: for real numbers x, *if ... then ...*

2. What proof would you choose? **direct** **indirect**
   Why?

3. What do you assume?

4. What conclusion do you wish to draw?

5. Complete the proof.

Prove for all integer m: *if* $5m + 4$ *is odd then* m *is odd.*

1. What proof would you choose? **direct** **indirect**
   Why?

2. What do you assume?

3. What conclusion do you wish to draw?

4. Complete the proof.

Prove that *half of an irrational number is irrational.*

1. Rephrase the proposition in the conditional form: for real numbers x, *if ... then ...*

2. What proof would you choose? **direct** **indirect**
   Why?

3. What do you assume?

4. What conclusion do you wish to draw?

5. Complete the proof.

Prove by contradiction that *if x is rational and y is irrational then $x + y$ is irrational.*

1. What are the assumptions?

2. What is the conclusion?

3. Assume the opposite of the conclusion in (2).

4. Based on (1) and (3), find a contradiction, or logical inconsistency.

5. Summarize the proof.

Someone told you that *if you multiply two odd numbers, the result is still odd.*

1. Do you think it's true? Why or why not?

2. If you think the proposition is true, prove it. If not, disprove it.

Someone told you that for any two integers m and n *if their product* mn *is even then either* m *is even or* n *is even.*

1. Do you think it's true? Why or why not?

2. Phrase the proposition as a conditional:*if ... then (... or ...).*

3. Rephrase the proposition in its contrapositive form.

4. Prove the propositioon in (3).