Discrete Mathematics Logic and Proof

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Logical Equivalence

Tautology and Contradiction

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Exercise. Is $(p \to q) \leftrightarrow (q \to p)$ a tautology, a contradiction, or neither?

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- **Example.** $(p \rightarrow q) \equiv (\neg p \vee q)$

Exercise

- Show that

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

Commonly Used Equivalences I

- Idempotent laws: $p \lor p \equiv p$ $\quad p \land p \equiv p$
- Associative laws: $(p \lor q) \lor r \equiv p \lor (q \lor r)$
  $(p \land q) \land r \equiv p \land (q \land r)$
- Commutative laws: $p \lor q \equiv q \lor p$ $\quad p \land q \equiv q \land p$
- Distributive laws: $p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$
  $p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$
- Identity laws: $p \lor F \equiv p$ $\quad p \lor T \equiv T$
  $p \land F \equiv F$ $\quad p \land T \equiv p$

Commonly Used Equivalences II

- Double negation law: $\neg\neg p \equiv p$
- Complement laws: $p \lor \neg p \equiv T$ $\quad \neg T \equiv F$
  $p \land \neg p \equiv F$ $\quad \neg F \equiv T$
- De Morgan's laws: $\neg(p \lor q) \equiv \neg p \land \neg q$
  $\neg(p \land q) \equiv \neg p \lor \neg q$
- Absorption laws: $p \lor (p \land q) \equiv p$ $\qquad p \land (p \lor q) \equiv p$
- Conditional identities: $p \rightarrow q \equiv \neg p \lor q$
  $p \leftrightarrow q \equiv (p \rightarrow q) \land (q \rightarrow p)$

Exercise:

- Prove the following De Morgan′s law:

$$\neg(p \land q) \equiv (\neg p) \lor (\neg q)$$