Discrete Mathematics
Logic and Proof

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Predicates and Quantifiers

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- If the domain is $\boldsymbol{Z}$ then $P(x)$ is *always* true.
- If the domain is $\boldsymbol{R}$ then $P(x)$ may be true or false, depending on the value of x.

Universal Propositions

$\forall x, P(x)$: “for all x, P(x)” or “for every x, P(x)”.

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- The symbol $\forall$ is a **universal quantifier**.
- The statement $\forall x, P(x)$ is called a **universally quantified proposition**.

Examples

Consider in the domain $\{1,2,3,4,5\}$ and the statements:

1. $\forall x, x^2 < 30$
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A counterexample is a particular case that makes the predicate of a universal statement false.

– Which of the propositions has a counterexample?

Existential Propositions

$\exists x, P(x)$: "There exists an x, such that P(x)".

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∃x, P(x): "There exists an x, such that P(x)".

∃x, P(x) asserts that P(x) is true for at least one possible value for x in its domain.

- The symbol ∃ is an **existential quantifier**.
- The statement ∃x P(x) is called an **existentially quantified proposition.**

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Consider in the domain $\{1,2,3,4,5\}$ and the statements:

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2. $\exists x, (x-3)^2 > 3.$

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An example is a particular case that makes the predicate of an existential proposition true.

– Which of the propositions has an example?