Discrete Mathematics
Logic and Proof

Pangyen Weng, Ph.D
Metropolitan State University

Nested Quantifiers

Review

$\forall x\ P(x)$: $P(x)$ is true for all x.

$\exists x\ P(x)$: $P(x)$ is true for at least one x.

Review

$\forall x\, P(x)$: $P(x)$ is true for all x.

$\exists x\, P(x)$: $P(x)$ is true for at least one x.

Example. Determine the truth value. n is an integer.

1. $\forall n\, (n^2 > 5)$
2. $\exists n\, (n^2 > 5)$

Examples

What are the meanings of the following proposition? And what are their truth values? Here x and y are in $\{0,1,2\}$ and $P(x, y): x = y$.

1. $\forall x \forall y\ (x = y)$

Examples

What are the meanings of the following proposition? And what are their truth values? Here x and y are in $\{0,1,2\}$ and $P(x, y): x = y$.

1. $\forall x \forall y \ (x = y)$
2. $\exists x \exists y \ (x = y)$

Examples

What are the meanings of the following proposition? And what are their truth values? Here x and y are in $\{0,1,2\}$ and $P(x, y): x = y$.

1. $\forall x \forall y\ (x = y)$
2. $\exists x \exists y\ (x = y)$
3. $\forall x \exists y\ (x = y)$

Examples

What are the meanings of the following proposition? And what are their truth values? Here x and y are in $\{0,1,2\}$ and $P(x, y): x = y$.

1. $\forall x \forall y\ (x = y)$
2. $\exists x \exists y\ (x = y)$
3. $\forall x \exists y\ (x = y)$
4. $\exists y \forall x\ (x = y)$

Alternating the Order

1. $\forall x \forall y \, P(x, y) \equiv \forall y \forall x \, P(x, y)$

Alternating the Order

1. $\forall x \forall y\, P(x, y) \equiv \forall y \forall x\, P(x, y)$
2. $\exists x \exists y\, P(x, y) \equiv \exists y \exists x\, P(x, y)$

Alternating the Order

1. $\forall x \forall y\, P(x, y) \equiv \forall y \forall x\, P(x, y)$
2. $\exists x \exists y\, P(x, y) \equiv \exists y \exists x\, P(x, y)$
3. $\exists x \forall y\, P(x, y) \not\equiv \forall y \exists x\, P(x, y)$