MATH 215 **DISCRETE MATHEMATICS** **SUMMER 2016**

Midterm Exam
May 31, 2016 Name:_________________________

This is a 3-hour exam. You may use a calculator and a letter-sized double-sided sheet of notes. No books, cellphones or other electronic devices are allowed. You must show all your work to receive full credit for each problem you solve. The highest score you may receive is 100 points. Submit your notes of Chapters 1–3 to receive 5 extra points.

1. (10 points) Use a truth table and determine if $(p \vee q) \to r$ and $(p \to r) \wedge (q \to r)$ are equivalent.

2. True or false?

   (a) (2 points) $((p \vee \neg q) \wedge q) \leftrightarrow p$ is a tautology. **True** **False**

   (b) (2 points) $\neg(p \vee q) \equiv (\neg p) \vee (\neg q)$ **True** **False**

   (c) (2 points) Consider for integers m and n, $\forall m \exists n (n = m + 2)$. **True** **False**

   (d) (2 points) $\neg(\forall x\, P(x)) \equiv \exists x\, \neg P(x)$. **True** **False**

   (e) (2 points) $\exists x \in \mathbf{Z}\, (x^2 = x)$ **True** **False**

3. (10 points) Prove that for any integer n, if n is even then $n^2 + n + 5$ is odd.

4. (10 points) Prove that for any real number x, if $x^2 + 1$ is irrational then x is irrational.

5. Answer the following questions.

   (a) (5 points) Draw the Venn diagram for the set $(A \cup B) \oplus C$.

   (b) (5 points) Let $A_i =$ the set of all integers from 0 to $i = \{0, 1, 2, \ldots, i\}$. Find $\bigcap_{i=3}^{10} A_i$.

6. Answer the following questions.

   (a) (5 points) Find the Cartesian product $A \times B$ for $A = \{-1, 0, 5\}$ and $B = \{x, y\}$. (Make sure you use adequate notations.)

   (b) (5 points) Evaluate $\lceil 3.1 + 5\lfloor -1.8 \rfloor \rceil$.

7. True or false?

   (a) (2 points) $\{\emptyset\} \subset \{1, 4, \emptyset, \{2, 3\}\}$ **True** **False**

   (b) (2 points) $f(x) = 2x + 1$, $f : \mathbb{N} \to \mathbb{N}$ is onto. **True** **False**

   (c) (2 points) $g(n) = n - 5$, $g : \mathbb{N} \to \mathbb{N}$ is well-defined. **True** **False**

   (d) (2 points) For any sets A, B and C, $A \cap (B \cup C) = (A \cap B) \cup C)$ **True** **False**

   (e) (2 points) $A = \{b, f, k, h, l\}$. Then $|P(A)| = 32$. **True** **False**

8. Answer the following questions.

   (a) (5 points) Find a_4 for the sequence defined by

$$a_0 = a_1 = 1, \ a_n = 5a_{n-1} - 3a_{n-2} \text{ for } n \geq 2.$$

   (b) (5 points) Expand and evaluate $\sum_{j=3}^{6} j(j+2)$.

9. Prove that for all integers $n \geq 0$, $\sum_{i=0}^{n} 3^i = \frac{3^{n+1}}{2} - \frac{1}{2}$.

(a) (1 points) What is the predicate $P(n)$?

(b) (2 points) **Base step.** Show that $P(0)$ is true.

(c) (2 points) **Inductive Step.** To prove that $P(k) \Rightarrow P(k+1)$, first phrase $P(k)$ and assume it.

(d) (5 points) Phrase $P(k+1)$. Show that $P(k+1)$ is true, assuming $P(k)$ is true.

10. (10 points) Let $P(n)$ be the statement that a gift of n dollars can be formed using just 5-dollar and 6-dollar gift certificates. Use strong induction to prove that $P(n)$ is true for $n \geq 20$.

11. Answer the following questions.

(a) Give a recursive algorithm so that the input is a positive integer n and the output is $\sum_{j=1}^{n} 4^j$. The algorithm must be recursive, it should not compute the sum using a closed form expression.

(b) Let $B = \{0, 1\}$. Give a recursive definition for the set S which is all binary strings in which there are no consecutive 1's. For example $101001 \in S$, but $010111 \notin S$.

(THE IS THE END OF THE TEST.)