MATH 215 **DISCRETE MATHEMATICS** **SUMMER 2015**

Exam I
June 9, 2015 Name:_____

This is a 3-hour exam. You may use a calculator and a letter-sized double-sided sheet of notes. No books, cellphones or other electronic devices are allowed. You must show all your work to receive full credit for each problem you solve. The highest score you may receive is 100 points.

1. Graph the Venn diagram for the following sets.

   (a) (3 points) $(A - B) \cup C$.

   (b) (3 points) $A \oplus (B - C)$.

2. Consider the set $A = \{1, 2, 3, \emptyset, \{x, y\}\}$. Determine if each of the following propositions is true or false.

   (a) (2 points) $\{x, y\} \subset A$. **TRUE** **FALSE**

   (b) (2 points) $\emptyset \in A$ **TRUE** **FALSE**

   (c) (2 points) $\{2, 3\} \subset A$ **TRUE** **FALSE**

   (d) (2 points) $\{x\} \subset A$ **TRUE** **FALSE**

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3. (7 points) Let $A_i = [0, i] = \{x : 0 \leq x \leq i\}$. Find $\bigcup_{i=1}^{n} A_i$ and $\bigcap_{i=1}^{n} A_i$.

4. (7 points) $f(x) = \dfrac{2x-1}{x+1}$. Find $f^{-1}(5)$.

5. (7 points) Let $A = \{a,\ c,\ e\}$ and $B = \{1,\ 3\}$. Find $A \times B$ and $B \times A$.

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6. Answer the following questions about functions. Explain your answer.

(a) (3 points) $f : \mathbb{R} \to \mathbb{R}$, $f(x) = \sqrt{x}$. Is f a function? **YES** **NO**

(b) (3 points) $f : \mathbf{Z} \to \mathbf{Z}$, $f(n) = |n - 3|$. Is f *one-to-one*? **YES** **NO**

(c) (3 points) $f : \mathbf{Z} \to \mathbf{Z}$, $f(x) = x + 6$. Is f *onto*? **YES** **NO**

(d) (3 points) Find a function that is one-to-one and onto. Do not use the identity function $f(x) = x$. Identify the domain and target.

(e) (3 points) Find a function that is one-to-one but **NOT** onto. Identify the domain and target.

(f) (3 points) Find a function that is onto but **NOT** one-to-one. Identify the domain and target.

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7. (7 points) Construct a truth table for the proposition $(p \oplus \neg q) \wedge (p \rightarrow r)$.

8. (7 points) Use a truth table and determine if the propositions $p \oplus (\neg q)$ and $p \leftrightarrow q$ are equivalent.

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9. Let $Q(x, y)$ be the statement "$y = x^2$" for all real numbers x and y. Find the truth values for the following. **Prove your answer.**

(a) (3 points) $Q(3, 9)$ **TRUE** **FALSE**

(b) (3 points) $Q(-3, -9)$ **TRUE** **FALSE**

(c) (3 points) $\forall x \exists y\, Q(x, y)$. **TRUE** **FALSE**

(d) (3 points) $\exists y \forall x\, Q(x, y)$. **TRUE** **FALSE**

(e) (3 points) $\forall y \exists x\, Q(x, y)$. **TRUE** **FALSE**

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10. (7 points) Is it true that for all real numbers x and y, $\lceil x - y \rceil = \lceil x \rceil - \lceil y \rceil$? Prove your answer.

11. (7 points) Prove that the product of two rational numbers is rational.

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12. (7 points) Prove that if $2x$ is irrational then x is irrational.

13. (7 points) Prove that if n^2 is an even number, then so is n.

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