MATH 215 **DISCRETE MATHEMATICS** **SUMMER 2015**

Exam II
July 14, 2015 Name:_____

This is a 3-hour exam. You may use a calculator and a letter-sized double-sided sheet of notes. No books, cellphones or other electronic devices are allowed. You must show all your work to receive full credit for each problem you solve. The highest score you may receive is 100 points.

1. (5 points) Find the first 5 terms for the sequence (b_n): $b_1 = b_2 = 1$, $b_n = 3b_{n-1} - b_{n-2}$ for $n \geq 3$.

2. (10 points) Prove that for all $n \in \mathbf{N}$, $\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$.

3. (5 points) Evaluate the summation $\sum_{j=3}^{8}(j^2 - j)$.

4. (10 points) Prove that for all positive integers $n \in \mathbf{N}$, $n^2 - n + 3$ is an odd number.

5. (5 points) How many different three-letter initials are there that begin with an A?

6. (5 points) In how many ways can a set of five letters be selected from the English alphabet?

7. (10 points) Suppose that a store offers gift certificates in denominations of 3 dollars and 7 dollars. Prove that when $n \geq 12$, you can form n dollars of gift using these gift certificates.

8. (5 points) There are 31 students in a discrete mathematics class at Metro State. Show that the class must have at least 16 male students or at least 16 female students.

9. (5 points) Use the principle of inclusion-exclusion to find the number of positive integers under $5,000$ that are not divisible by either 2 or by 3.

10. Suppose someone takes out a home improvement loan for \$150,000. The annual interest on the loan is 6% and is compounded monthly. The monthly payment is \$1,200. Let a_n denote the amount owed at the end of the nth month. The payments start in the first month and are due the last day of every month.

    (a) (5 points) Give a recurrence relation for a_n. Don't forget the base case.

    (b) (5 points) Suppose that the borrower would like a lower monthly payment. How large does the monthly payment need to be to ensure that the amount owed decreases every month?

11. (5 points) Give a recursive algorithm to compute the sum of the cubes of the first n positive integers. The input to the algorithm is a positive integer n, the output is $\sum_{j=1}^{n} \left(\frac{1}{j^2}\right)$. The algorithm must be recursive, it should not compute the sum using a closed form expression.

12. Suppose that 100 people enter a drawing and that different winners are selected at random for first, second, and third prizes. Jack and Jill are among the 100 people in the drawing.

    (a) (5 points) What is the probability that both Jack and Jill both win prizes?

    (b) (5 points) What is the probability that Jack wins the first prize and Jill wins the second prize?

13. (5 points) Find the probability of each outcome when a biased die is rolled, if the probability of rolling a 3 is 10%, the probability of rolling a 4 is 30%, and the probability of rolling each of the other 4 numbers on the die is equally likely.

14. (5 points) At a university, 40% of students have completed their writing requirements, 35% have completed their math requirements, and 16% have completed both. What is the probability that a randomly chosen student has completed **math or writing** requirements?

15. (5 points) A Minneapolis construction company has hired 30 new workers. 5 of the 30 workers are of ethnic minority. If work assignments are made randomly, what is the probability that the three worst assignments are all given to workers of ethnic minority?

16. Five poker cards are randomly drawn from a deck to form a poker hand. Find the probability of each of the following.

    (a) (2 points) The poker hand contains precisely three heart cards.

    (b) (2 points) The poker hand contains an Ace, a King, and two Queens.