MATH 215 **DISCRETE MATHEMATICS** **FALL 2015**

Exam I Part A
Due: 9 am, 10/1/2015

Name: Weng

1. (5 points) What is the probability that a five-card poker hand contains two Aces and two Queens?

$$\frac{C(4,2)\cdot C(4,2)\cdot C(44,1)}{C(52,5)}$$

2. (5 points) A Minneapolis construction company has hired 30 new workers. 5 of the 30 workers are of ethnic minority. If work assignments are made randomly, what is the probability that two of the three worst assignments are given to workers of ethnic minority?

$$\frac{C(5,2)\cdot C(25,1)}{C(30,3)}$$

3. (3 points each) Answer the following questions about functions. Explain your answer.

(a) $f : \mathbb{R} \to \mathbb{R}$, $f(x) = \dfrac{x-5}{x+8}$. Is f a function? YES NO

$f(-8)$ is undefined

(b) $f : \mathbf{Z} \to \mathbf{Z}$, $f(n) = n + 7$. Is f *one-to-one*? YES NO

(c) $f : \mathbf{Z} \to \mathbf{Z}$, $f(x) = 2x - 3$. Is f *onto*? YES NO

There is no x such that $f(x) = 0$

(d) Find a function that is onto but **NOT** one-to-one. Identify the domain and target.

$f: \mathbb{R} \to \mathbb{Z}$ $f(x) = \lfloor x \rfloor$

Exam I Part B
October 1, 2015

Name: Weng

This is a 3-hour exam. Attach your Part A to the paper at time of submission. You may use a calculator and a letter-sized double-sided sheet of notes. No books, cellphones or other electronic devices are allowed. You must show all your work to receive full credit for each problem you solve. The highest score you may receive is 100 points.

1. Consider the following intervals for $i \in \mathbf{N}$, $A_i = [0, i) = \{x : 0 \leq x < i\}$.

   (a) (5 points) Write down A_3, A_4, A_5, A_6 and A_7.

   $A_3 = [0,3)$

   $A_4 = [0,4)$

   $A_5 = [0,5)$

   $A_6 = [0,6)$

   $A_7 = [0,7)$

   (b) (5 points) Find $\bigcup_{i=3}^{7} A_i$ and $\bigcap_{i=3}^{7} A_i$.

   $\bigcup_{i=3}^{7} A_i = A_3 \cup A_4 \cup \cdots \cup A_7 = [0,7)$

   $\bigcap_{i=3}^{7} A_i = A_3 \cap A_4 \cap \cdots \cap A_7 = [0,3)$

2. Consider the set $A = \{1, 2, 3, \emptyset, \{x, y\}\}$. Determine if each of the following propositions is true or false.

   (a) (2 points) $\{x, y\} \subset A$. $\{x,y\} \in A$ TRUE **FALSE**

   (b) (2 points) $\emptyset \in A$ **TRUE** FALSE

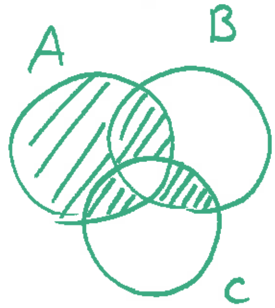
   (c) (2 points) $\{2, 3\} \subset A$ **TRUE** FALSE

   (d) (2 points) $\{x\} \subset A$ $\{x\} \notin A$ & $\{x\} \not\subset A$ TRUE **FALSE**

   (e) (2 points) $|A| = 6$ $|A| = 5$ TRUE **FALSE**

page total:

3. (5 points) Draw the Venn diagram for $A \oplus (B \cap C)$.



4. (5 points) How many bit strings of length 15 contain at least 3 zeros?

E = containing 3 zero S = Sample space

$\overline{E}$ = containing 2, 1 or no zeros

$|\overline{E}| = C(15,2) + C(15,1) + C(15,0)$

$|E| = |S| - |\overline{E}|$

$= 2^{15} - C(15,2) - C(15,1) - C(15,0)$

5. Assume one person out of 2,000 is infected with disease X. A test was developed to detect disease X. According to scientists, those who have the disease will definitely test positive in the test, but those who do NOT have the disease also have a 2% chance of testing positive in the test.
Let X be the event of having disease X and T be the event of testing positive. Find the following.

(a) (2 points) $P(T|X)$, the probability of testing positive *for someone with disease X.*

1

(b) (2 points) $P(X \cap T)$, the probability of having disease X and testing positive.

$\frac{1}{2000} \cdot 1$

(c) (2 points) $P(T|\overline{X})$, the probability of testing positive *for someone without disease X.*

0.02

(d) (2 points) $P(\overline{X} \cap T)$, the probability of *not* having disease X and testing positive.

$\frac{1999}{2000} \cdot 0.02$

(e) (2 points) $P(T)$, the probability of testing positive.

$\frac{1}{2000} + \frac{1999}{2000} \cdot 0.02$

(f) (2 points) $P(X|T)$, the probability of having disease X *given a positive testing result.*

$$\frac{\frac{1}{2000}}{\frac{1}{2000} + \frac{1999}{2000} \cdot 0.02}$$

page total:

6. Consider $f(x) = \lceil \frac{x-1}{3} \rceil$, $f : \mathbf{R} \to \mathbf{Z}$.

(a) (5 points) Is f one-to-one? Explain your answer.

No. For example $f(1) = \lceil \frac{0}{3} \rceil = 0$

$f(0) = \lceil \frac{-1}{3} \rceil = 0$

(b) (5 points) Is f onto? Explain your answer.

Yes. For any $y \in \mathbf{Z}$, $f(3y+1) = \lceil \frac{3y+1-1}{3} \rceil = y$.

7. (5 points) Let $A = \{a, x, y\}$ and $B = \{0, 3, 8\}$. Write down $A \times B$.

$$A \times B = \{ (a,0), (a,3), (a,8), (x,0), (x,3), (x,8), (y,0), (y,3), (y,8) \}$$

8. Let f and g be functions from $\mathbf{R}$ to $\mathbf{R}$, $f(x) = (x+1)^3$ and $g(x) = 3x - 5$.

(a) (5 points) Evaluate $(f \circ g)(3)$.

$g(3) = 3 \cdot 3 - 5 = 4$

$f(g(3)) = f(4)$

$= (4+1)^3 = 5^3 = 125$

(b) (5 points) Find a formula for $g^{-1}(x)$.

$y = g(x) = 3x - 5 \Rightarrow x = \frac{y+5}{3}$

So $g^{-1}(x) = \frac{x+5}{3}$

page total:

9. Suppose that 500 people enter a drawing and that different winners are selected at random for first, second, and third prizes. Jack and Jill are among the 500 people in the drawing.

   (a) (4 points) What is the probability that ONLY Jill wins a prize?

   $|S| = C(500, 3)$

   E_1: Jill wins a prize , $|E_1| = 1 \cdot C(497, 2)$

   $$p(E_1) = \frac{1 \cdot C(497, 2)}{C(500, 3)}$$

   (b) (4 points) What is the probability that Jack wins the first prize and Jill wins the second prize?

   $$\frac{1}{500} \cdot \frac{1}{499}$$

10. A 2-letter initial consists of the first letters in one's first and last names in capital. For example, BW = the 2-letter initial for Ben Weng.

   (a) (5 points) How many different two-letter initials can be generated from the 26 alphabets?

   $26 \cdot 26 = 26^2 = 676$

   (b) (5 points) There are 2,000 new students entering Metro State this fall. Show that at least 3 of them have the same two-letter initials.

   $\left\lceil \frac{2000}{676} \right\rceil = 3$ by Pigeonhole Principle