

**3A. Curves in Space and Their Tangents**

What is a position vector  $\mathbf{r}(t)$ ? What are its *component functions* and what is the curve traveled by  $\mathbf{r}$ ?

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Evaluate:  $\lim_{t \rightarrow 0} \left\langle \tan(1 - t^2), \frac{e^t - 1}{t}, \frac{\sin t}{2t} \right\rangle.$

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Is  $\mathbf{r}(t) = (t^2)\mathbf{i} + (\sec t)\mathbf{j} - (5 \sin t)\mathbf{k}$  continuous at  $t = \pi/2$ ? Explain your answer.

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$\mathbf{r} = (\cos 2t)\mathbf{i} + (\sin 2t)\mathbf{j} + (5t)\mathbf{k}$ . Find its velocity, speed and acceleration at  $t = \pi/2$ .

**3B. Integrals of Vector Functions Part 1**

How do you evaluate *indefinite integrals* for vector functions?

Evaluate

$$\int \left( (t\sqrt{1+t^2})\mathbf{i} + (6\sqrt[3]{t})\mathbf{j} + (e^t \sin(e^t + 1))\mathbf{k} \right) dt.$$

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How do you evaluate *definite integrals* for vector functions?

Evaluate

$$\int_0^1 \left( (t\sqrt{1+t^2})\mathbf{i} + (6\sqrt[3]{t})\mathbf{j} + (e^t \sin(e^t + 1))\mathbf{k} \right) dt.$$

**3B. Integrals of Vector Functions Part 2**

Suppose we do not know the position  $\mathbf{r}$  of a moving object, but we do know its acceleration is  $\mathbf{a}(t) = (9 \sin 3t)\mathbf{i} - (9 \cos 3t)\mathbf{j}$  and that initially the object departed from the point  $(1, 2, 0)$  with velocity  $3\mathbf{j} + \mathbf{k}$ . Solve for  $\mathbf{r}$ .

Write down the formula(s) for the ideal projectile motion. Write down at least 3 conclusions that can be derived from the result of this analysis.

**3C. Arclength**

What is the formula for calculating the arclength of  $\mathbf{r}(t)$  from  $t = a$  to  $t = b$ ?

Set up the integral for the arclength of  $\mathbf{r} = e^t\mathbf{i} + 2t\mathbf{j} + t^2\mathbf{k}$  from  $t = 0$  and  $t = 2$ . This is not an integral that can be calculated by hand, but find a way to estimate it.

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Find the unit tangent vector for  $\mathbf{r} = e^t\mathbf{i} + 2t\mathbf{j} + t^2\mathbf{k}$  at  $t = 1$ .

**3D. Curvature and Normal Vectors**

What does curvature measure? What is the formula for calculating the curvature of a parametric curve?

Find the curvature of  $\mathbf{r} = t\mathbf{i} + 2t\mathbf{j} + t^2\mathbf{k}$  at  $t = 1$ .

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What is the principal normal vector  $\mathbf{N}$  for a parametric curve? What is the formula for calculating it?

Find the principal normal vector  $\mathbf{N}$  of  $\mathbf{r} = t\mathbf{i} + 2t\mathbf{j} + t^2\mathbf{k}$  at  $t = 1$ .

**3E. Tangential and Normal Components Normal Components of Acceleration**

What is a TNB-frame? Write down the formulas for  $\mathbf{T}$ ,  $\mathbf{N}$  and  $\mathbf{B}$ .

What is a torsion? Write down the formulas for the torsion for a parametric curve  $\mathbf{r}$ .

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Consider for  $\mathbf{r} = \langle \cos 3t, \sin 3t, 4t \rangle$  at  $t = 0$ .

1. Find  $\mathbf{T}$ ,  $\mathbf{N}$  and  $\mathbf{B}$ .
2. Find its torsion.