Discrete Mathematics
Properties of Integers

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Division of Integers

Quotient and Remainder

Let a be an integer and n be a **positive** integer. If

$$a = n \cdot q + r, \qquad 0 \leq r < n$$

Then we call this a valid division of a by n. We also call:

q: **quotient** notation: $q = a \operatorname{div} n$

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Example. What is the quotient and what is the remainder?

$$103 = 5 \cdot 20 + 3$$

More Examples

Find the quotient and remainder of:

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2. -1274 divided by 6

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1. 9513 divided by 12.
2. -128 divided by 9

Divisibility

Let a and b be two integers. We say a is *divisible by* b iff

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Example. True or false? (a) $6|\ 1494$ (b) $7|1494$

Example

Prove: *If* $3|m$ *and* $3|n$ *then* $3|(7m - 8n)$