

# **Discrete Mathematics**

## **Sets and Functions**

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# Introduction to Functions

# Functions

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 $Y$ : the **target** of  $f$

› The **range** of  $f$  is the set of those elements that are actually hit in the target, i.e.

$$\{y: (x, y) \in f, \text{ for some } x \in X\}$$

## Example

Consider  $X = \{0,1\}$ ,  $Y = \{a, b, c, d\}$ , and a function  
 $f = \{(0, b), (1, a)\}$

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3. Express  $f$  in an **arrow diagram**.

## Expressing Functions by Formulas

We often write functions in formulas. When doing so, do not forget to identify the domain and the target.

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$$f(x) = \sqrt{x}, \quad f: \mathbb{R}^+ \rightarrow \mathbb{R}^+.$$

**Example 2.**

$$f(x) = \frac{4}{x^2 + 1}, \quad f: \{0, 1, 2, 3, 4\} \rightarrow \mathbb{R}$$

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