Discrete Mathematics
Sets and Functions

Pangyen Weng, Ph.D
Metropolitan State University

Properties of Functions

Injective Functions

A function $f: X \rightarrow Y$ is ***one-to-one*** or ***injective*** if

$$x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2).$$

That is, f maps different x to different y.

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Examples.

1. $f(x) = x^2, f: R \rightarrow R$ is NOT one-to-one.
2. $f(x) = x^2, f: R^+ \rightarrow R^+$ is one-to-one.

Exercise

Name a function that is one-to-one, and a function that is not one-to-one.

- Make sure you identify the domain and the target.
- Additional challenge: can you use floor or ceiling functions?

Surjective Functions

A function $f: X \to Y$ is ***onto*** or ***surjective*** if the range of f is equal to the target Y. That is, for every $y \in Y$, there is an $x \in X$ such that $f(x) = y$.

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Examples.

1. $f(x) = |x|$, $f: Z \to Z$ is NOT onto.

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Examples.

1. $f(x) = |x|, f: Z \rightarrow Z$ is NOT onto.
2. $f(x) = |x|, f: N \rightarrow N$ is onto.

Exercise

Name a function that is onto, and a function that is not onto.

- Make sure you identify the domain and the target.
- Additional challenge: can you use quadratic or cubic function?

Bijective Functions

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2. $f(x) = 2x, f: N \to N$ is NOT bijective.

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2. $f(x) = 2x, f: N \to N$ is NOT bijective.

Remark. If function $f: A \to B$ is a bijection and A is finite, then $|A| = |B|$.