Topic 01 Sets and Subsets

What is a set? What is an *element* of a set?

Write down in set notation the set of the 5 smallest positive integers. What is its *cardinality*?

What is the *empty set*?

What is a subset of a given set? Write in notation: $\{1, 2, 3\}$ *is a subset of* $\mathbf{N}$.

What do these sets denote? $\mathbf{N}$, $\mathbf{Z}$, $\mathbf{Q}$ and $\mathbf{R}$.

Describe the set of even numbers.

Topic 02 Set of Sets

Can a set be both an element and a subset of a bigger set? Make an example or explain why not.

Consider the set $S = \{1, 2, 3, \{\emptyset\}, \{1, 2\}\}$.

1. How many elements does S have? What are they?
2. Is $\{1, 2\}$ an element or a subset?
3. Is $\emptyset$ an element or a subset?
4. Is $\{\emptyset\}$ an element or a subset?

What is a power set? Write down the power set of $A = \{2, 3, 5\}$.

If a set S has 8 elements, how many elements does $P(S)$ have?

Topic 03 Intersection and Union

What is the universal set U?

Let $U = \{1, 2, 3, 4, 5, 6, 7\}$ and $A = \{1, 3, 5\}$. Draw a Venn diagram for them.

Let $A = \{1, 2, 4, 8, 16\}$ and $B = \{2, 4, 6, 8, 10\}$. Draw their Venn diagram. Find $A \cap B$ and $A \cup B$.

Let $A_1 = \{1, 2, 3, 4, \ldots\}$ and $A_2 = \{2, 3, 4, \ldots\}$. Find $A \cap B$ and $A \cup B$.

$A_i = \{i, i+1, i+2, \ldots\}$ for $i = 1, 2, 3, \ldots$. Find $\bigcap_{i=1}^{5} A_i$ and $\bigcup_{i=1}^{5} A_i$.

Topic 04 More Set Operations

Let A and B be two sets.
What is $A - B$, the *difference* from A to B? Draw its Venn diagram.

What is $A \oplus B$, the symmetric difference between A and B? Draw its Venn diagram.

What is $\overline{A}$, the *complement* of A? Draw its Venn diagram.

Draw the Venn diagram for $A \oplus (B - C)$.

Let $U = \{1, 2, \ldots, 9, 10\}$, $A = \{2, 4, 6, 8, 10\}$ and $B = \{3, 6, 9\}$. Find the following.

1. $A - B$.

2. $A \oplus B$.

3. $\overline{A}$.

Topic 05 Cartesian Products

What is an order-pair? How do we denote an ordered-pair?

What is the Cartesian product of two sets A and B? Write down the Cartesian product for $A \times B$ if $A = \{0, 1\}$ and $B = \{\text{true, false}\}$.

If A has 5 elements and B has 8 elements, how many elements does $A \times B$ have?

Write down the Cartesian product $A \times B \times C$ if $A = \{\text{burger, sandwich}\}$, $B = \{\text{fries, chips}\}$ and $C = \{\text{coke, sprite, fanta}\}$.

Write down A^3 if $A = \{T, F\}$.

Topic 06 Introduction to Functions

How do you record functions using ordered pairs?

What's the domain, target and range of a function?

Let $X = Y = \mathbf{R}$ and $f = \{(0, 5), (1, 8), (2, -1.4)\}$.

1. Find $f(0)$, $f(1)$ and $f2)$.

2. What is the domain of f? Target? Range?

3. Express the function using an arrow diagram.

Make a function that is expressed in a formula. Do not forget to identify the domain and target.

Find the floor function and the ceiling function for the values: 3.1, -4.8, 2 and -6.

Topic 07 Properties of Functions

What is a one-to-one function? Name a function that is one-to-one and a function that is NOT. Remember to identify the domain and the target.

What is an onto function? Name a function that is onto and a function that is NOT. Remember to identify the domain and the target.

What is a bijective function? Name a function that is bijective and a function that is NOT. Remember to identify the domain and the target.

Topic 08 Inverse and Composite Functions

What is an inverse function?

$X = \{x, y\}$, $Y = \{1, 3, 4, 8\}$ and $f = \{(x, 3), (y, 8)\}$. Find f^{-1}.

$g : \mathbf{R} \to \mathbf{R}$, $g(x) = 3x - 1$. Find a formula for g^{-1}.

f, g, h are functions from $\mathbf{R}$ to $\mathbf{R}$,

$$f(x) = x + 5,\ g(x) = x^2,\ h(x) = \lceil x/3 \rceil.$$

Evaluate the following.

1. $(f \circ g)(2)$ and $(g \circ f)(2)$.

2. $(g \circ h)(5)$ and $(h \circ g)(5)$.

3. $(f \circ f \circ f)(-4)$.